SUPPLEMENTARY MATERIAL

*This appendix has been provided by the authors to give readers additional background reading*

# Supplement to

**Grunau G, Linn S. Detection and Diagnostic Overall Accuracy Measures of Medical Tests. Rambam Maimonides Med J 2018;9 (4):e0027. doi:10.5041/RMMJ.10351**

## DERIVATION FROM EQUATION 2

The overall detection accuracy is:

$$\text{(Eq. 2) Overall detection accuracy} = \frac{a+d}{a+b+c+d} = \frac{\frac{a}{a+c}(a+c)+\frac{b}{b+d}(b+d)}{a+b+c+d}$$

$$= \frac{sensitivity(a+c)+specificity(b+d)}{(a+c)+(b+d)}$$

Dividing the numerator and denominator by $b+d$ we get:

$$= \frac{\frac{sensitivity(a+c)}{(b+d)}+\frac{specificity(b+d)}{(b+d)}}{\frac{(a+c)+(b+d)}{b+d}} = \frac{sensitivity\frac{(a+c)}{(b+d)}+specificity\frac{(b+d)}{(b+d)}}{\frac{a+c}{b+d}+\frac{b+d}{b+d}}$$

$$= \frac{\frac{sensitivity(a+c)}{(b+d)}+specificity}{\frac{a+c}{b+d}+1} = \frac{sensitivity*x+specificity}{x+1}$$

Where:

$a$=true positives in the study population (number of sick persons who tested positive);

$b$=false positives in the study population (number of not-sick persons who tested positive);

$c$=false negatives in the study population (number of sick persons who tested negative);

$d$=true negatives in the study population (number of not-sick persons who tested negative);

$x$=disease prevalence odds; and

lower-case letters denote “a study population.”

## UNDERSTANDING PREVALENCE

Only when the proportion of sick persons, $prevalence_s$ in a specific study is identical to the prevalence in the patient population, that is, $prevalence_{Table1} = prevalence$, is the detection accuracy identical to the diagnostic accuracy.

$$\text{(Eq. 2) Overall detection accuracy} = \frac{a+d}{a+b+c+d} = \frac{sensitivity * x + specificity}{x+1}$$

$$= \frac{sensitivity * \frac{prevalence_s}{1-prevalence_s} + specificity}{\frac{prevalence_s}{1-prevalence_s} + 1}$$

$$= \frac{sensitivity * \frac{prevalence_{Table1}}{1-prevalence_{Table1}} + \frac{specificity * (1-prevalence_{Table1})}{1-prevalence_{Table1}}}{\frac{prevalence_{Table1} + 1 - prevalence_{Table1}}{1-prevalence_{Table1}}}$$

$$= sensitivity * prevalence_{Table1} + specificity * (1-prevalence_{Table1})$$

Only if $prevalence_{Table1}$=$prevalence$ do we get Equation 11:

$$sensitivity * prevalence + specificity * (1-prevalence) = \frac{A+D}{A+B+C+D}$$

$$= \text{overall diagnostic accuracy (Eq. 11)}$$

Where:

A=true positives in the patient population, i.e., number of persons who tested positive who were sick;

B=false positives in the patient population, i.e., number of persons who test positive and were not sick

C=false negatives in the patient population, i.e., number of persons who test negative and were sick;

D=true negatives in the patient population, i.e., number of persons who test negative who were not sick; and

upper-case letters denote “a patient population.”

## CALCULATING PREVALENCE

Calculating prevalence-specific clinical data for Table 3 based on the sensitivity and specificity for 1000 patients. Note that the prevalence for each population is $A+C/1000$ (5%, 50%, and 90%).

### Population I

| | Gold Standard | | Total |
|---|---|---|---|
| | $S_{POS}$ | $S_{NEG}$ | |
| Clinical Test | | | |
| $T_{POS}$ | $A$ = 30 | $B$ = 86 | *116* |
| $T_{NEG}$ | $C$ = 20 | $D$ = 864 | *884* |
| Total | *50* | *950* | *1000* |

Prevalence=50/1000=0.05

Overall diagnostic accuracy=894/1000=0.894

### Population II

| | Gold Standard | | Total |
|---|---|---|---|
| | $S_{POS}$ | $S_{NEG}$ | |
| Clinical Test | | | |
| $T_{POS}$ | $A$ = 300 | $B$ = 45 | *345* |
| $T_{NEG}$ | $C$ = 200 | $D$ = 455 | *655* |
| Total | *500* | *500* | *1000* |

Prevalence=500/1000=0.5

Overall diagnostic accuracy=755/1000=0.755

### Population III

| | Gold Standard | | Total |
|---|---|---|---|
| | $S_{POS}$ | $S_{NEG}$ | |
| Clinical Test | | | |
| $T_{POS}$ | $A$ = 540 | $B$ = 9 | *549* |
| $T_{NEG}$ | $C$ = 360 | $D$ = 91 | *451* |
| Total | *900* | *100* | *1000* |

Prevalence=900/1000=0.9

Overall diagnostic accuracy=631/1000=0.631

Where

$S_{POS}$=sick;

$S_{NEG}$=not sick;

$T_{POS}$=positive test;

$T_{NEG}$=negative test;

## ESTIMATING THE DIFFERENCE BETWEEN THE TWO MEASURES OF OVERALL ACCURACY

By applying Equation 14 to Table 1 and Table 2, we obtain equations for the difference in the magnitude of the overall detection accuracy and the overall diagnostic accuracy:

Difference of overall accuracy measures in Table 1 versus Table 2

$$= \text{overall diagnostic ability} - \text{overall detection accuracy}$$

$$= [sensitivity * prevalence + specificity * (1 - prevalence) - sensitivity * prevalence_{Table1}$$

$$+ specificity * (1 - prevalence)]$$

$$= sensitivity * (prevalence - prevalence_{Table1}) - specificity * (prevalence - prevalence_{Table1})$$

$$= (sensitivity - specificity) * (prevalence - prevalence_{Table1})$$

Thus, for a test with a given difference between the sensitivity and specificity, the difference between the two overall accuracy measures is dependent solely on the difference in the prevalence estimates in a specific study $prevalence_{Table1}$ versus the true general patient population prevalence.